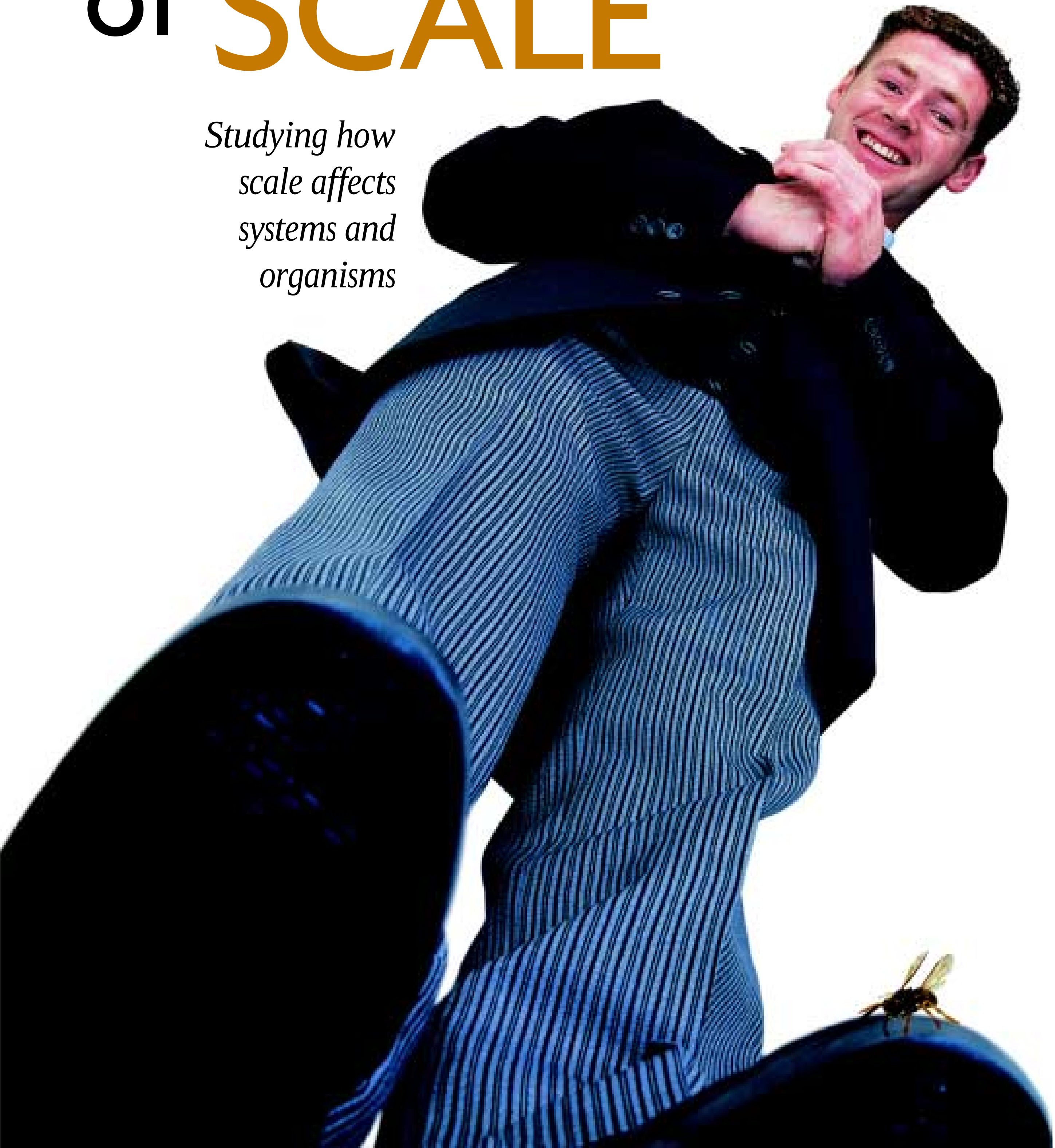
A Sense of SCALE

Studying how scale affects systems and organisms



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In the Dr. Seuss (1954) classic *Horton Hears a Who!*, Horton the elephant discovers an entire other world on a speck of dust. This tiny world was oblivious to the larger world, and vice versa, until an inhabitant of one made contact with an inhabitant of the other. Although a fictional story, a similar dividing line between worlds of different sizes does exist.

Suppose we consider tiny insects—for many of them the surface tension of water in a pond creates a strong, concrete-like barrier to penetration. However, for larger creatures, such as humans, surface tension is essentially negligible—humans sink right through the water's surface. If tiny insects were to fall from a 10-story building, air resistance would be sufficient enough to keep their falling speed from reaching dangerous levels, and they would walk away from the landing without a second thought (if insects even have first thoughts). Air resistance couldn't protect relatively huge creatures (such as humans) who would likely never survive such a fall. Although existing in the same physical world, the size difference between a tiny insect and a human implies that interactions in their respective worlds are very different indeed.

The concept of how scale affects systems and organisms is central to many science disciplines and serves as a unifying theme identified by Project 2061 as important for all students (AAAS 1993). By the end of eighth grade this benchmark indicates that, "Students should know that properties of systems that depend on volume, such as capacity and weight, change out of proportion to properties that depend on area, such as strength or surface processes" (AAAS 1993, p. 278). By the end of twelfth grade, "Students should know that because different properties are not affected to the same degree by changes in scale, large changes in scale typically change the way that things work in physical, biological, or social systems" (AAAS 1993, p. 279). The *Principles and Standards for School Mathematics* also emphasize the importance of scaling concepts and the effects of scale changes on a problem situation (NCTM 2000).

In spite of the centrality of scale to many science disciplines, the pressure to cover specific content in a course may make it easy to overlook this unifying theme. But its importance should not be ignored. For example, as students begin investigating objects as small as the nanometer range (Jones, Superfine, and Taylor 1999), their ability to consider the importance of scale becomes even more critical. One of the fastest growing areas of science is nanotechnology—research and development of materials at the nanometer scale. Developments in nanoscale technology have resulted in the construction of nanometer-sized motors and construction of materials one atom at a time. The challenge for teachers is to help students develop an understanding of how tiny nanometers are and how interactions differ for objects at this minute scale. We have developed the following activities that intentionally focus on the importance of scale and its effects.

Scientific notation and size

In order for students to appreciate the vast differences in size of various biological or physical systems, they need a firm grasp of scientific notation and corresponding sizes. Although the following activity uses distance as a measure, it can be modified for time (for example, in a geology class) or any other measurement that spans several orders of magnitude and is important to the curriculum.

An activity focusing on the powers of 10 can help students understand scientific notation and begin to internalize the meaning of the notation in terms of size. One possible resource is found at *www.wordwizz.com/pwrsof10.htm.* This website includes a series of images seen from 10^{25} m, a distance of approximately one billion light years, down to 10^{-16} m, deep inside a carbon atom proton. This activity shows students what the universe would look like at various scales and can be done as a whole class activity choreographed by the teacher using a data projector or by individual students or small groups in a computer lab.

We prefer to begin by clicking on the 10^0 m image of a man asleep; this familiar image serves as a good starting point. Clicking on the +1 button zooms out by a factor of 10, leaving behind a box to show where the previous full-screen image boundary was located. This helps students make the conceptual leap from one image to the subsequent one that is 10 times more distant. Students or the teacher can continue to zoom out by a factor of 10 with each click by using the +1 button until the limits of the universe are reached at 10^{25} m. Students can also explore the "more info" button at any stage if desired. At this point, the teacher can return back to the 10^0 m reference image and begin the inward journey down into the skin of the man.

If done as a whole class activity, it may be helpful for the teacher to pause and discuss certain scales. For example, the similarities at very large and very small scales are striking—they depict mostly empty space. The teacher can also discuss scales that will appear in subsequent lessons, such as the large cosmic scales that are relevant to Newton's theory of universal gravitation. Providing students with a table similar to Figure 1, p. 24 helps them connect the scientific notation, name, and corresponding size.

Scale on a clothesline

Next, students can participate in an activity to help them visualize the distances of various powers of 10 m and make the connection between symbolic representation and reality. This activity is modified from one presented by Laubach, Royce, and Holzer (2000). Students hang a clothesline (or string) across the room and use small spring clothespins to clip index cards (labeled "0 m" and "1 m") at a distance of 1 m from each other (Figure 2, p. 24). Alternatively, they can tape the cards on the hallway wall 1 m apart, or use chalk on an outside sidewalk to mark these points.

Teachers should allow space on the negative side of 0 for students to use in their deliberations of the numbers.

FIGURE 1

Powers of 10.

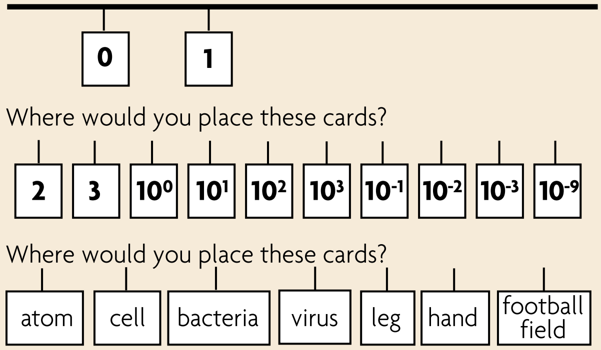
Teachers should use a table similar to this to help students connect the scientific notation, name, and corresponding size.

PREFIX	POWER OF 10	NUMBER	NAME
meter	10^0	1.0	one
deci-	10^{-1}	0.1	tenth
centi-	10^{-2}	0.01	hundredth
milli-	10^{-3}	0.001	thousandth
micro-	10^{-6}	0.000 001	millionth
nano-	10^{-9}	0.000 000 001	billionth

FIGURE 2

Scale on a clothesline.

Students are asked to clip cards onto a clothesline (or string) in the approximate proper location.



Students are asked to clip the cards shown in Figure 2 to the clothesline in the approximate proper location. The card with 10^{-9} m is included because the subsequent lesson will focus on viruses, which are nanometer (10^{-9} m) in size, but teachers could include or exclude any powers of 10 that may or may not be relevant to the lesson as long as a large-enough range is present to have a visual effect on the students.

Usually the 2 m and 3 m cards are relatively easy to place. Teachers should allow the students to discuss the card placements and make corrections (as students explain why) after each card is clipped to the clothesline. Often, 10^0 gives some trouble, thus a discussion of the meaning of something raised to the 0 power ensues. The positive powers of 10 give varying degrees of trouble, depending on the class. Ideally, the 10^1 m card should have enough room to be placed (many rooms are not large enough, but hallways or sidewalks work well). The approximate and relative position of the cards is stressed, rather than the precise measurement.

The goal of this introductory activity is to get students to think conceptually about the relative scale of numbers. The 10^2 m and the 10^3 m cards cannot usually be placed because of the distances involved, so students are asked to estimate where they think the cards should go. Approximate distances, such as "the end of the parking lot," are used for 10^2 m or a specific road intersection near the school is used for 10^3 m. Students note this on a sheet of paper and tape it to the card to show where it would go if the string was long enough.

What is 10^{-1}?

The negative powers of 10 often are difficult for students to understand. Students tend to place them on the left side of the 0—the negative side of the number line—as opposed to between 0 and 1. Teachers should remind students that 10^{-1} m means one tenth of a meter (and is still greater than 0). This activity reinforces the meaning of scientific notation and involves physically placing numbers as they discuss the conceptual meaning of the relative order.

Relative size

Once students have some understanding of scientific notation and the order and magnitude of the numbers, they can begin to conceptualize the relative size of different objects and relate the size of common objects to the powers of 10 notation. The goal is for students to develop quick conceptual links for relative size. Therefore, teachers should pick objects familiar to students. The human leg is about the length of a meter, and a hand width is about a tenth of a meter. Students often hold misconceptions about the relative size of small objects and think viruses are larger than bacteria, for example. Having students think about whether or not an atom is larger than a virus helps them understand relative size. A second set of cards (Figure 2) can be used to help students visualize relative size along the number line they have created. Can they match virus to nanometer? Or leg to meter?

Logarithmic scales

Many scientific measurements are logarithmic in nature, such as pH, decibels, the Richter scale for earthquake intensity, or star brightness. Students often have difficulty grasping the meaning of logarithmic scales and making relative comparisons between them; for example, understanding that a magnitude 7 earthquake generates wave amplitudes 10 times higher than a magnitude 6. The previous clothesline activity naturally leads into a discussion on why logarithmic scales are necessary for certain measurements. Students quickly see there simply isn't room to hang the 10^2 m and higher powers of 10 in a classroom. One possible solution to the problem of large powers of 10 not conveniently fitting on the clothesline is to space each power of 10 evenly on the line. This way very large and very small numbers can be represented, but teachers should emphasize to students that the distances are not the actual distances represented by the numbers.

Using examples related to the course content, like earthquake intensity, teachers should ask students to describe the difference between a magnitude 6 and magnitude 7 earthquake. Teachers might uncover the powers of 10 to remind students what the "6" and "7" really mean to emphasize the increase by a factor of 10. This also can lead to a discussion of the grid lines on log-linear or log-log graph paper (if that is something students will use in the class). A concrete representation of logarithms such as this is sometimes helpful because the mathematical treatment of logarithms can leave many students confused, and as a result measurements based on logarithms are often misunderstood.

Scaling effects

The previous foundational activities help prepare students for further work specific to the content area of the class. One primary reason that scale is important in many cases is that the surface area-to-volume ratio changes at different scales, and therefore processes tied to surface area will tend to predominate at small scales over those tied to volume. Some examples below are given to stimulate educators' thinking about how this might apply to a course, but there are certainly others that may be useful.

Biology

The field of allometry is a specialty within biological sciences that focuses on scaling effects. Teachers could relate scale to processes of cellular absorption and waste elimination. Because these are surface area phenomena, if a cell became too large, it would have too little surface area relative to its volume for nutrients to be absorbed and waste products eliminated, thereby effectively limiting the viable size of a cell. Or students might consider absorption in the context of organ systems and use scaling effects to explain why the small intestine of humans is so long, whereas for some worms the digestive tract is essentially a straight tube through the body. A small creature will have enough surface area from a straight small intestine for nutrient absorption, whereas a large creature needs to increase the surface area with lots of twists and turns.

Similarly, teachers could discuss the process of oxygen transfer in the lungs (a surface area phenomenon) as they discuss size and function of alveoli. Small creatures such as protozoa do not need such an elaborate system for the intake of oxygen because they have enough surface area relative to their volume to simply absorb oxygen through their bodies. Other phenomena related to strength (a cross-sectional area phenomenon) and size or mass (volume phenomena) could be emphasized. Because the diversity of life spans over 21 orders of magnitudes, the size of something certainly affects its interactions with the world.

Physics/physical science

Processes such as evaporation and heat transfer depend on the surface area-to-volume ratio, as does air resistance and other frictional effects. For example, small animals such as ants have a large surface area relative to their mass, and so air resistance prohibits them from ever attaining a dangerous speed when falling from a height. Larger creatures have relatively more mass, and hence, not enough surface area to significantly slow their fall.

As an example of how evaporation is related to size, recently a lizard the size of a U.S. dime, the smallest of all 23 000 species of reptiles, birds, and mammals, was found in the Dominican Republic (A little lizard 2002). Scientists hypothesize that this is reaching the physiological limit of small size, because if the creature were any smaller the relatively large surface area would cause such a lizard to die by evaporation through its skin.

The size of structures such as bridges and skyscrapers are constrained partly because the scaling effects of the strength of materials is dependent primarily on cross sectional area considerations. The physics of microscale and nanoscale is likely to become increasingly important as ever-tinier mechanical components are fabricated (Livermore 2001).

These examples are merely a few of a wide array of possibilities for helping students understand the scaling effects of something being larger or smaller. This central, unifying theme across many science disciplines can be taught in ways that supplement the specific content of a particular course, rather than seeming to be an extra concept to fit into an already-tight curriculum. Students' imaginations can be stimulated as they envision a different world—a world of very different scale from the normal human scale we experience every day. ■

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